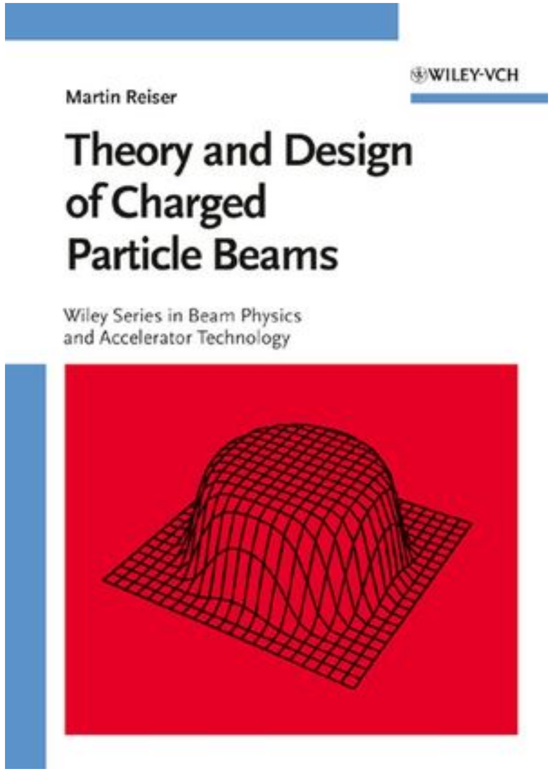
Science & Technology Facilities Council

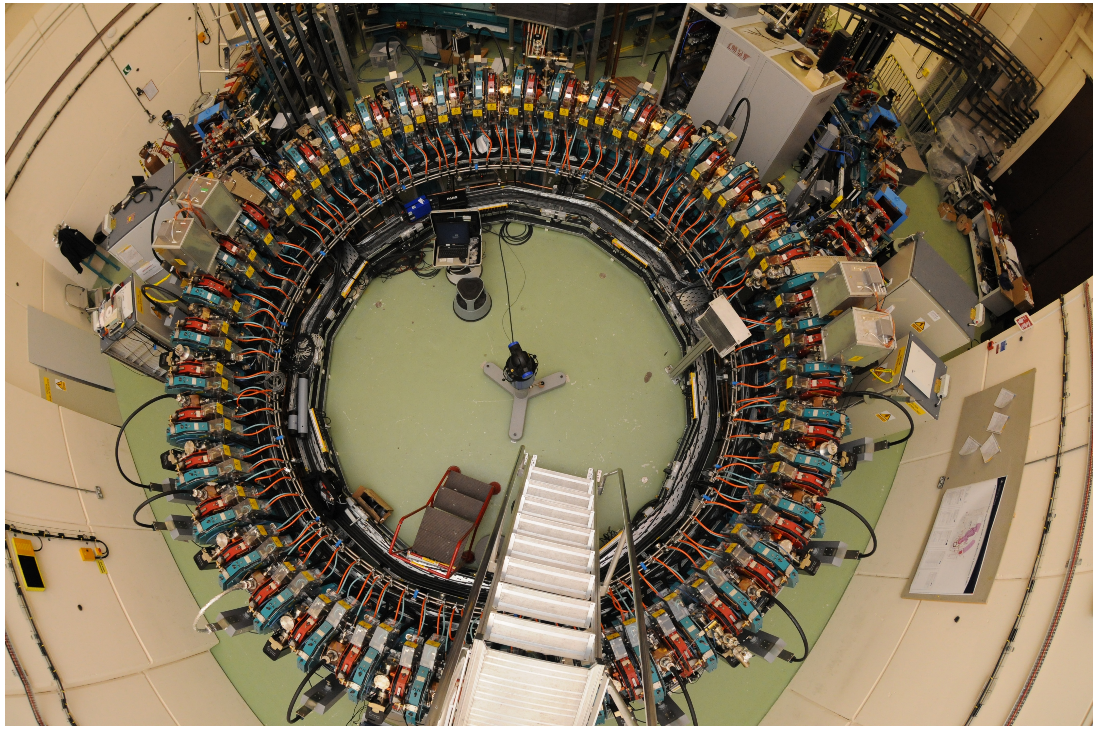
The study of scaling FFAGs in a multipole ion trap

David Kelliher (ISIS/RAL/STFC) on behalf of the IBEX team,

with thanks to H. Okamoto and his group.

How we study accelerators





We typically employ theory, simulation, experimental beam physics studies.

Analogue experiment

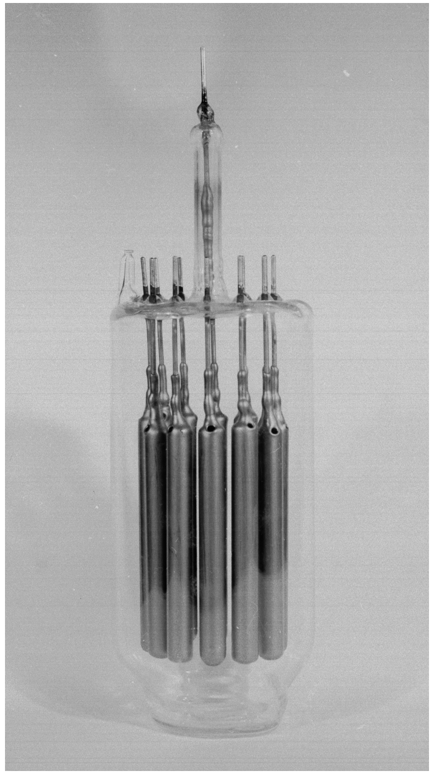


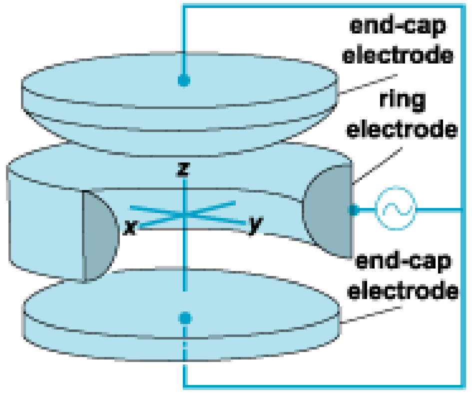
Fig. 1: The PS analogue model

[M. Barbier, 1956]

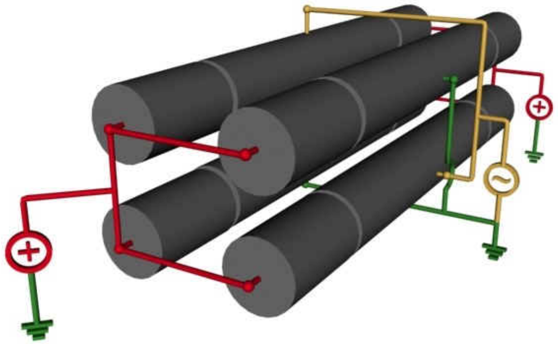
- Analogue experiments offer another tool to study accelerator physics.

Paul traps

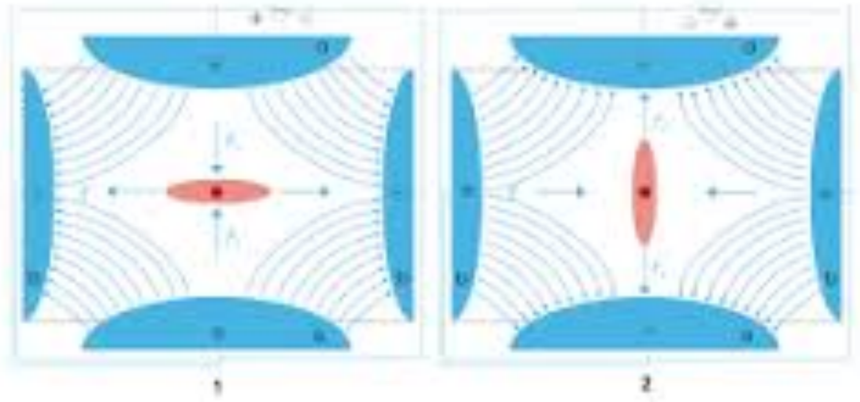
- Paul trap: Non-neutral plasma trap in which an RF field confines the plasma radially via strong focusing. DC voltages confines the plasma longitudinally.
- Unlike a quadrupole channel, the plasma is trapped in-situ with the confining waveform applied in time. This is analogous to the co-moving frame in the beam case.



Paul trap, Wolfgang Paul (1953).



Linear Paul Trap (1989).



Strong focusing.

Paul trap applications

Mass spectroscopy, quantum computing research …

Comparison of transverse dynamics (1)

Accelerator case.

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} k_2(s) \left(x^2 - y^2\right) + \frac{q}{p_0 \beta_0 c \gamma_0^2} \phi_{sc}$$ where $$k_2 = \frac{1}{B\rho} \frac{\delta B_y}{\delta x}$$

lattice

space charge

Paul trap case.

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} k_2^P(\tau) \left(x^2 - y^2\right) + \frac{q}{mc^2} \phi_{sc}$$ where $$k_2^P(\tau) = \frac{2qV_q(\tau)}{mc^2 r_0^2}$$

where in both cases the space charge potential ϕ_{sc} is found by solving the Poisson equation.

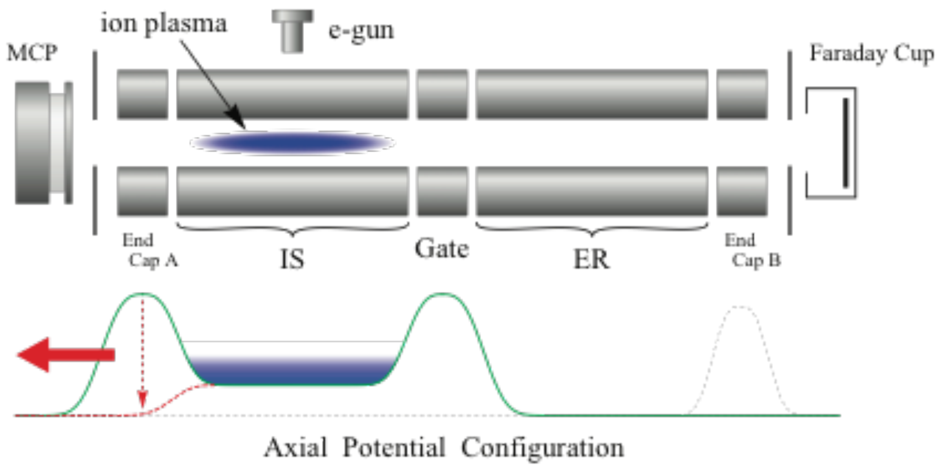
Beam physics studies in LPTs

Advantages include

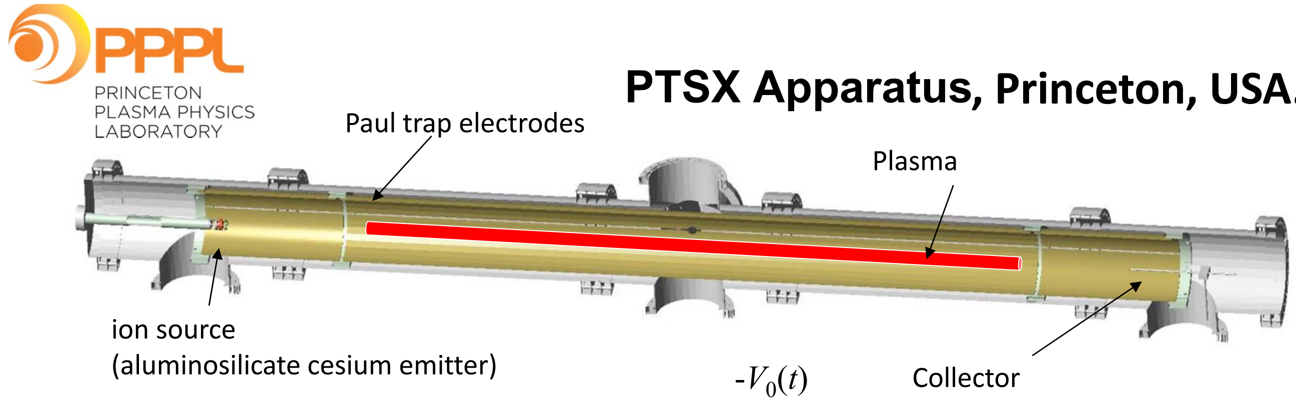
- **Flexibility**: The lattice structure, working point and plasma density can be selected within a wide range.
- **Compactness**: Traps are typically ~10cm long and, together with ancillary equipment, require just a few m^2 of laboratory space.
- **Cost**: Construction and running costs are orders of magnitude lower than carrying out experiments on typical accelerators.
- **Availability**: Typically, time allocated for beam physics experiments on accelerators is limited. No such limit applies to plasma traps.

Devices to date

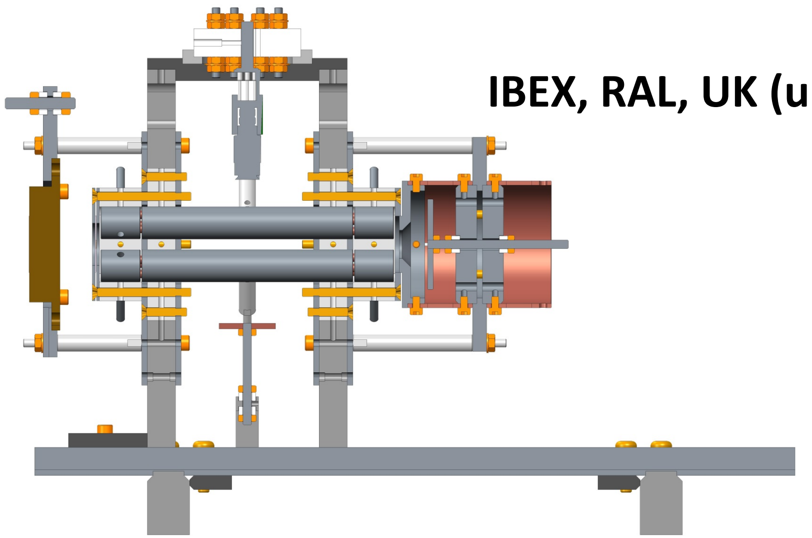
SPOD (various devices), Hiroshima, Japan.



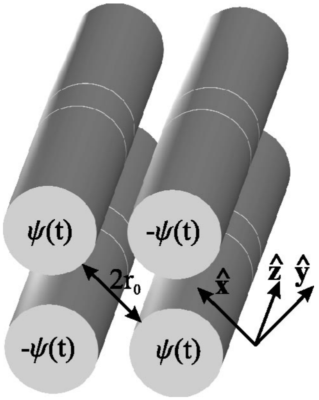
PTSX Apparatus, Princeton, USA.



IBEX, RAL, UK (under commissioning).



Aarhus, Denmark.



Linear Paul trap basics.

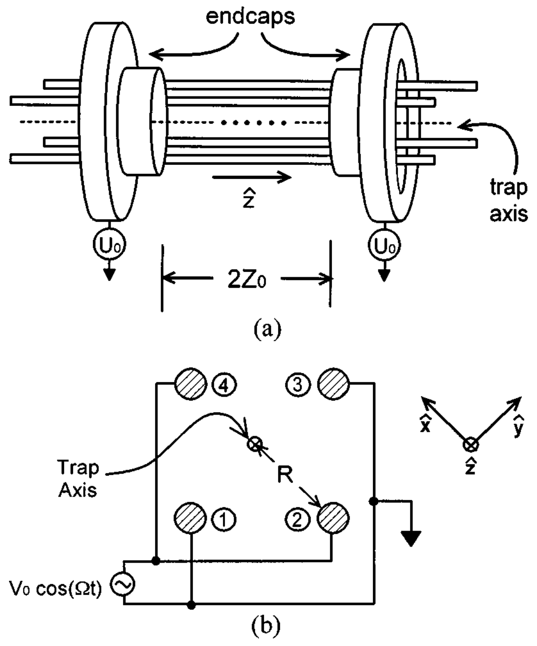


FIG. 1. Linear Paul trap (a) side view and (b) axial view. A string of trapped ions is shown schematically in (a). For clarity, the endcaps are not shown in (b). The trap electrodes are labeled 1, 2, 3, and 4. The trap axis defines the z-axis, and the origin of the z axis is centered between the two endcaps.

RF and DC potential terms.

$$V_{rf}(x,y,t) \approx V_q \left(\frac{x^2 - y^2}{R^2} \right)$$

$$U_{DC}(x,y,z) = \frac{\xi U_0}{Z_0^2} \left[z^2 - \frac{1}{2}(x^2+y^2) \right]$$

Eqn. of motion (Mathieu equation).

$$\ddot{u}_i + [a_i + 2q_i \cos\Omega t] \frac{\Omega}{4} u_i = 0$$

Stability parameters (a,q).

$$a_{x,y} = -\frac{1}{2} a_z = -\frac{8\xi e U_0}{m Z_0^2 \Omega^2}$$

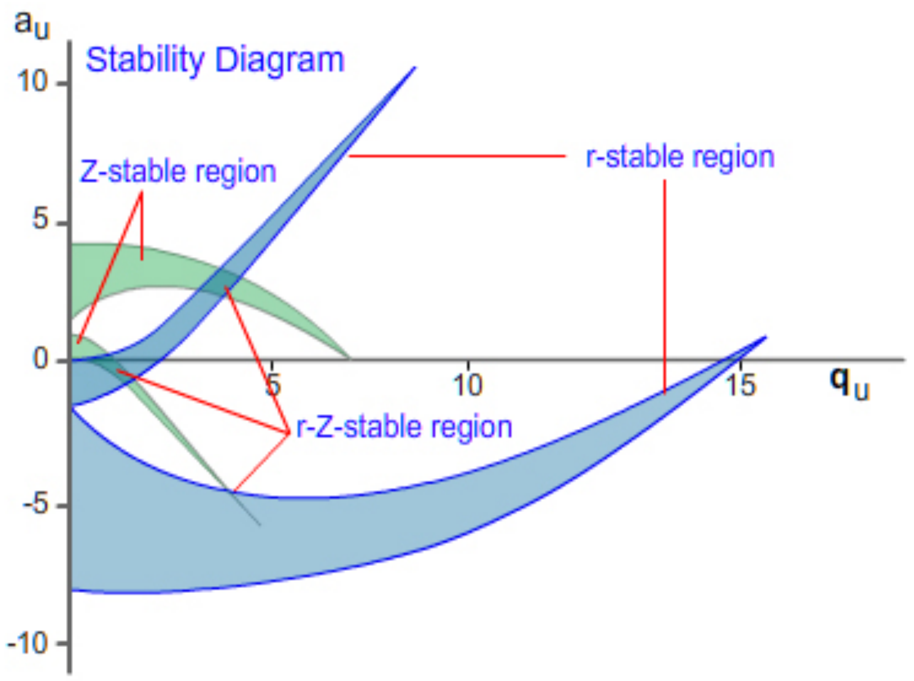
$$q_x = -q_y = -\frac{4eV_q}{m r_0^2 \Omega^2}$$

Secular/micromotion (betatron/envelope) .

$$u_i(t) = u_0 \cos\omega_o t \left(1 + \frac{q_i}{2} \cos\Omega t \right)$$

$$\omega_0 \approx \frac{\Omega}{2} \sqrt{a_i + \frac{q_i}{2}}$$

$$\nu = \frac{\omega_0}{\Omega}$$

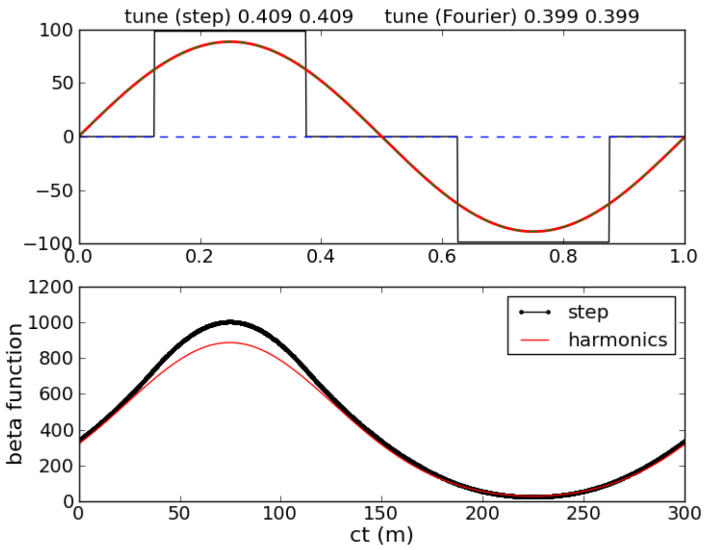


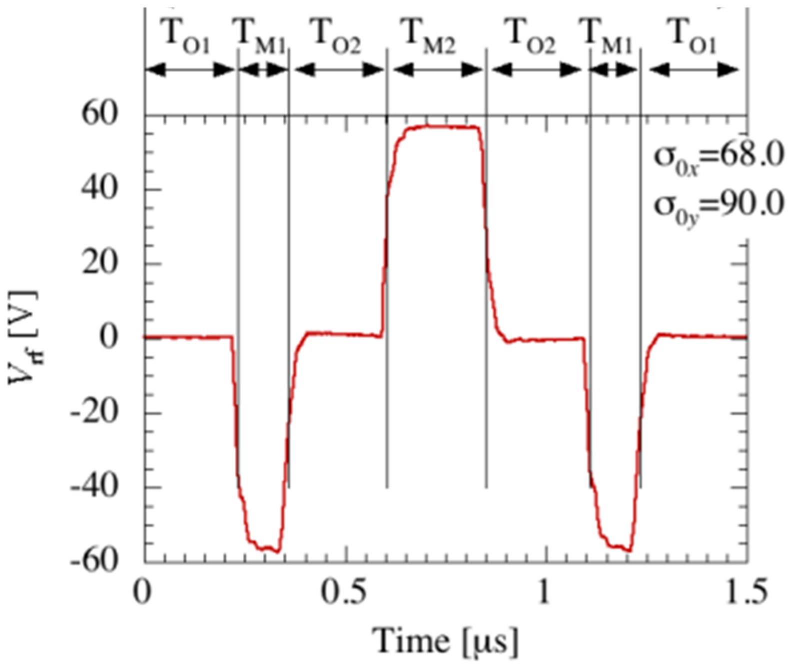
For example assume Ar+, 1MHz RF, 5mm radius. ν = 0.25 if Vq = +/- 72 V.

[D. J. Berkeland et al, 1998]

Lattice flexibility

Many lattice configurations can be realised . One constraint is the bandwidth of the RF amplifier.





Triplet lattice [Okamoto, IPAC14]

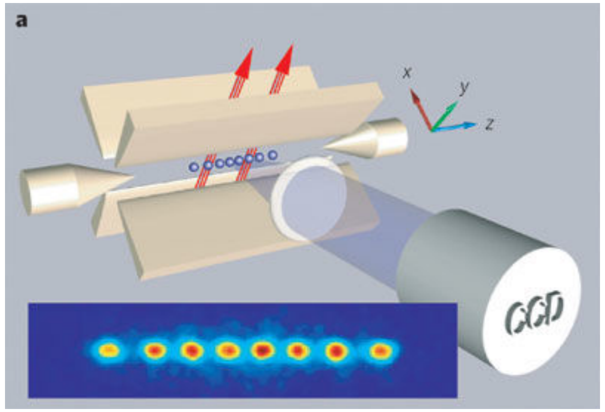
Space charge

- Assuming a stationary plasma the tune depression is given by

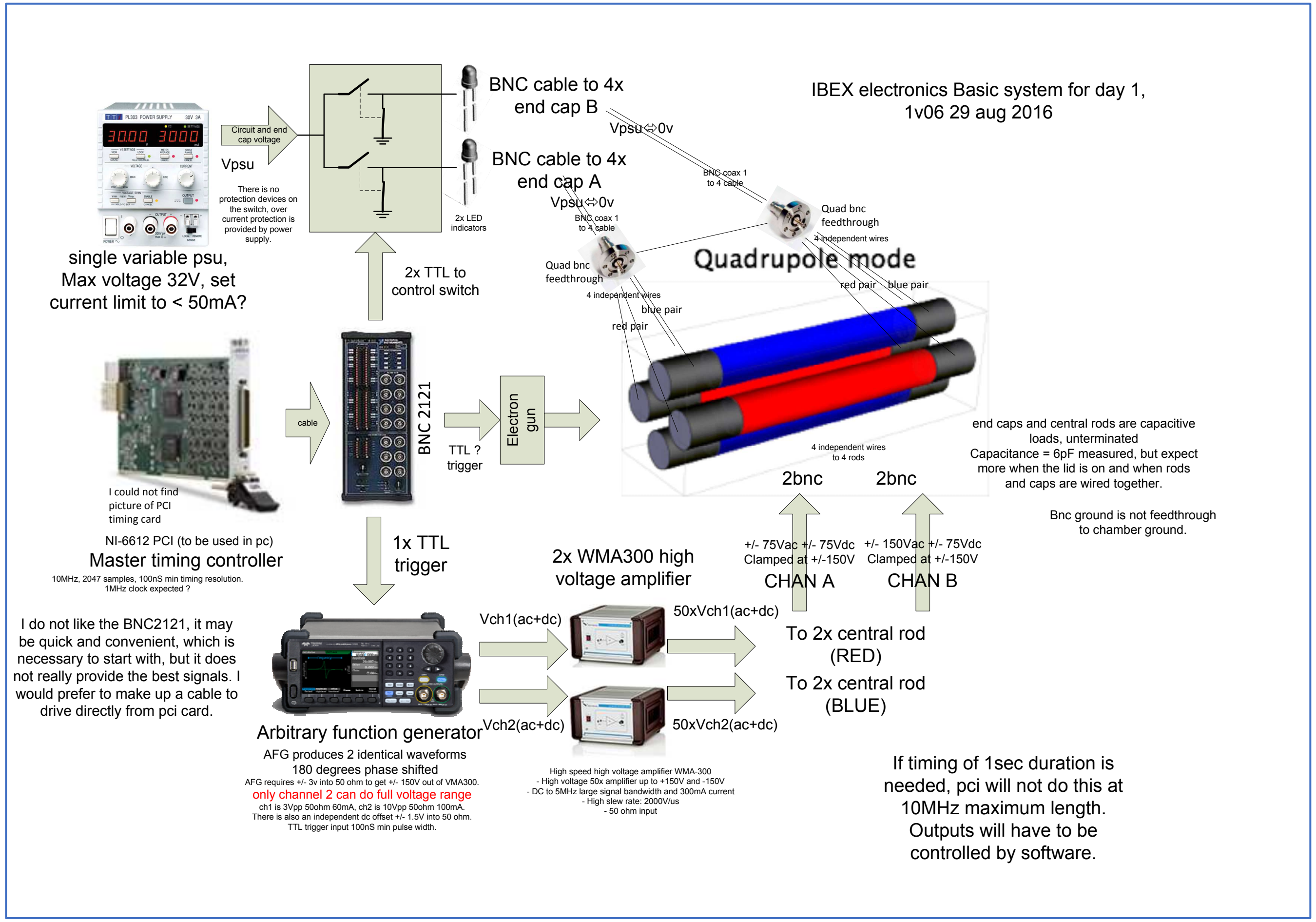
$$\frac{\nu}{\nu_0} = \sqrt{1 - \frac{1}{1 + (2/Nr_p)(k_B T_\perp / mc^2)}}$$

- In SPOD/IBEX temperatures typically around 0.1-0.5eV and up up to 10^7 ions can be stored (ν/ν_0 ~ 0.85-0.9). This is more than enough to simulate typical high intensity rings.
- Note: In the zero temperature limit, accessible by Doppler laser cooling, $\nu \rightarrow 0$. In this limit the plasma frequency equals the bare secular frequency (betatron tune).

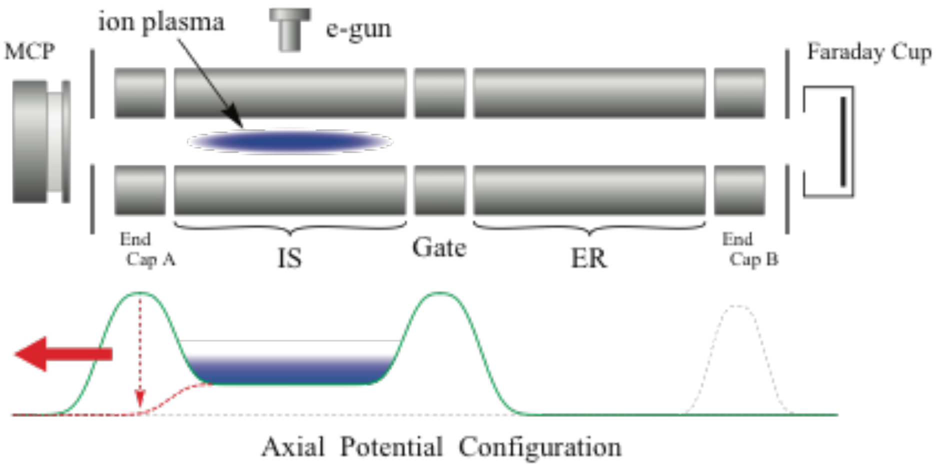
$$\omega^2 = \omega_0^2 - \frac{\omega_p^2}{2}$$

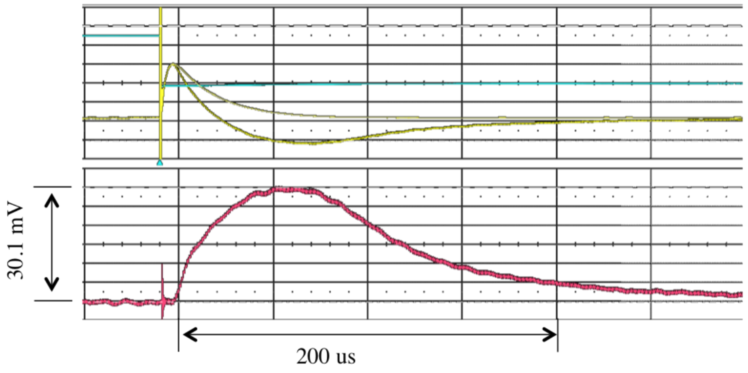


IBEX experimental setup



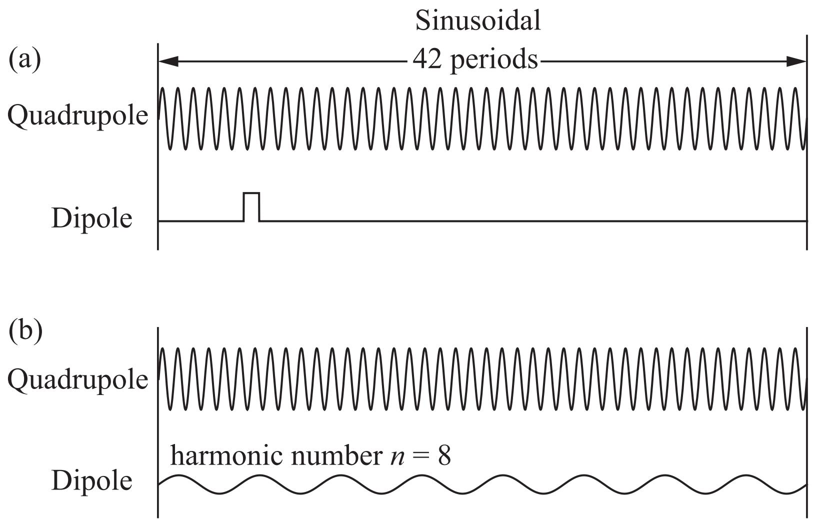
Stage 1 hardware setup (Schematic: A. Baird)

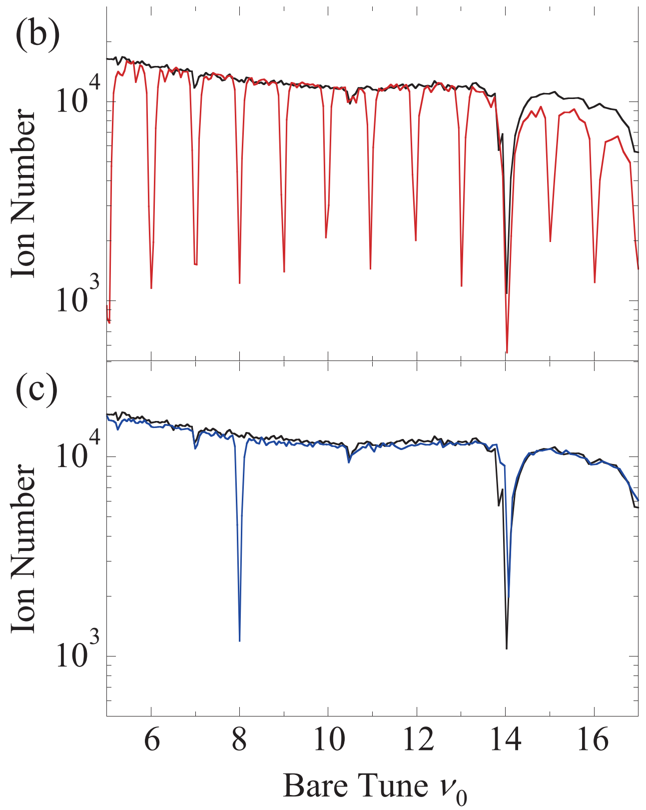


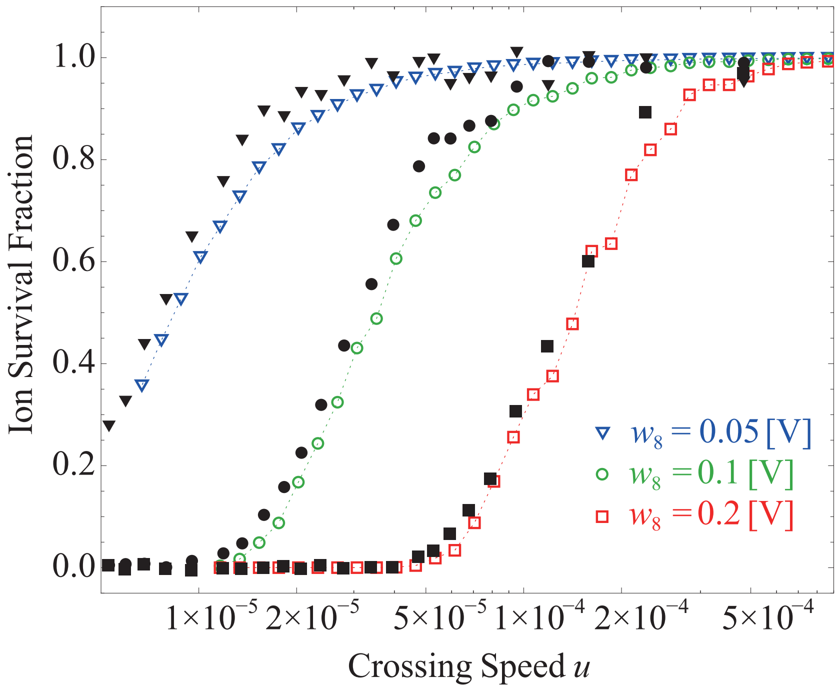


Amplify and integrate FC signal

Integer crossing in non-scaling FFAGs.







- Introduce integer resonance by adding perturbation waveform. Ramp tune through resonance by varying voltage.
- Measure number of surviving ions. Compare with theory and numerical simulation.

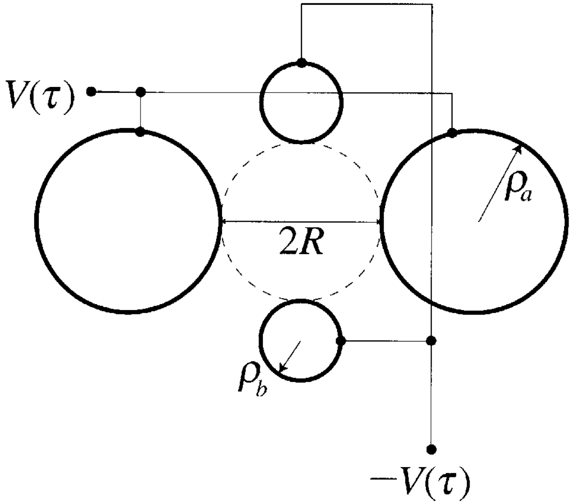
[K. Moriya et al, (2015)]

Scaling FFAG studies.

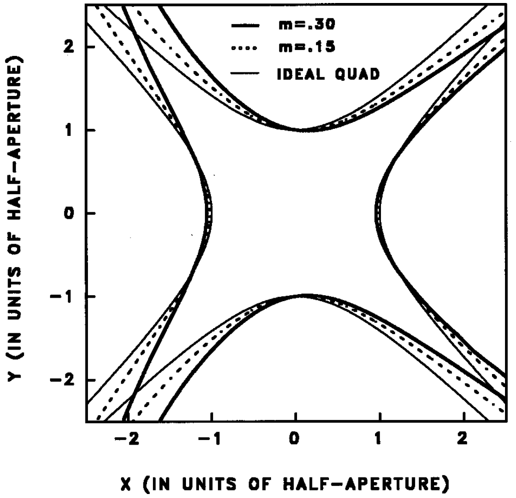
- Aim: study lattice nonlinearities with space charge.

- Construct a Paul trap in which the quadrupole term is still dominant. Nonlinear terms, at least up to octopole are added – either by adding extra rods or by shaping the pole face of the quadrupole rods.
- Such a trap would allow us to study lattices with strong nonlinearities together with a substantial space charge tune shift.

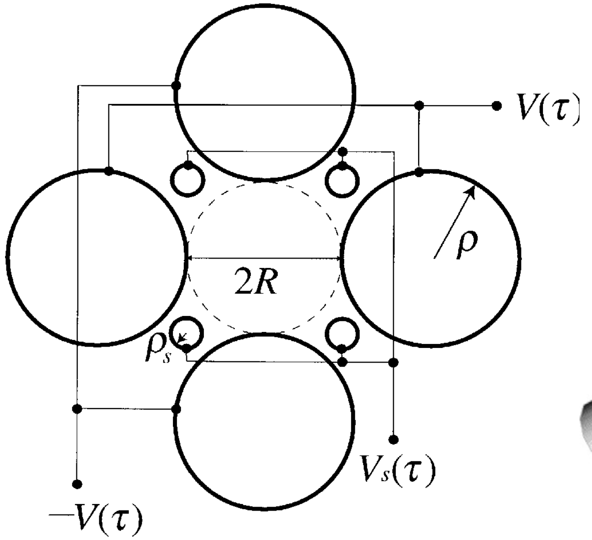
Introducing nonlinear components



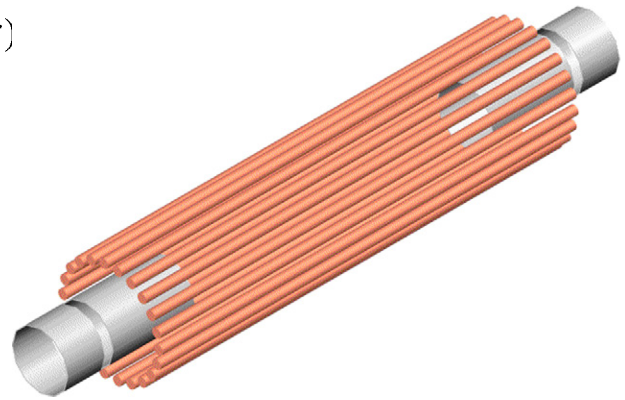
[Okamoto, 2002]



[Sarma, 1999]

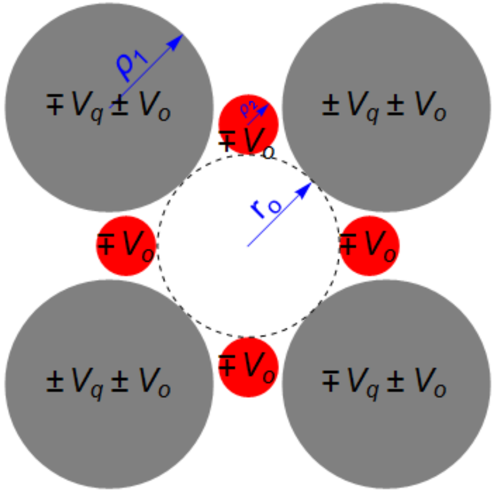


[Okamoto, 2002]



[Asvany, 2009]

Multipole trap



- Maintain radius of main rods to inscribed radius radio $\rho_1/r0 \cong 1.15$ to ensure good quadrupole focusing.
- Add “subrods” to control sextupole and octupole components.
- For any rod geometry, solve the Laplacian numerically to calculate the coefficients a_n, φ_n in the multipole expansion

$$F(r,\theta) = \sum_{n=1}^{\infty} a_n \left(\frac{r}{r_0}\right)^n \cos(n\theta + \varphi_n)$$

Hamiltonian: multipole case

Hamiltonian including normal quadrupole, sextupole and octupole terms (ignoring skew terms).

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2}k_2\left(x^2 - y^2\right) + \frac{1}{6}k_3\left(x^3 - 3xy^2\right) + \frac{1}{24}k_4(x^4 - 6x^2y^2 + y^4)$$

where

Accelerator case $$k_n = \frac{1}{B\rho}\frac{\delta^{n-1}B}{\delta x^{n-1}}$$

Paul trap case $$k_n^P(\tau) = \frac{n!a_n q V_n(\tau)}{mc^2 r^n}$$

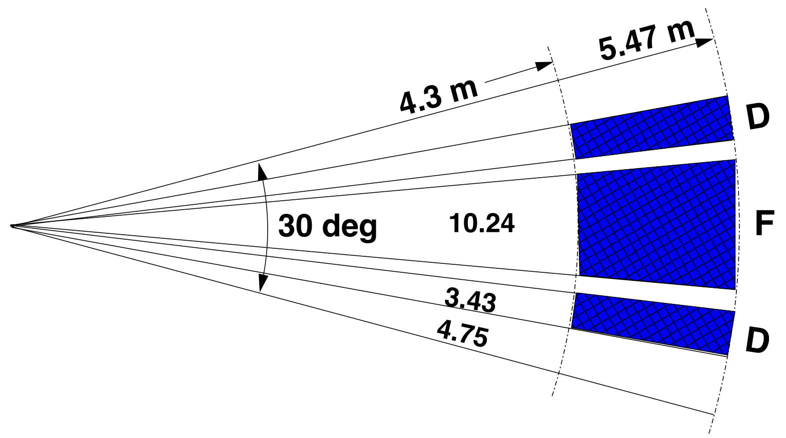
Scaling FFAG case $$B = B_0\left(\frac{r}{r_0}\right)^{\kappa} \longrightarrow k_n = \frac{\kappa!}{(n-1)!\rho r^{n-1}(\kappa - n + 1)!}$$

Linear optics translation

1. Convert scaling FFAG to a straight quadrupole channel.
   - Including the quadrupole component from the scaling field may not be sufficient. Also need to take into account the effect of edge focusing.
2. Convert quadrupole channel to LPT.
   - In order to preserve tune, scale quadrupole strength k_2 with 1/L^2 and convert to RF voltage. Note, transfer matrix for thin lens drift-quad-drift

$$\begin{pmatrix} 1 & L/2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -k_2 L & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & L/2 \end{pmatrix} = \begin{pmatrix} 1 - k_2 L^2/2 & \frac{L}{2} + \frac{L}{2}(1 - k_2 L^2/2) \\ -k_2 L & 1 - k_2 L^2/2 \end{pmatrix}$$

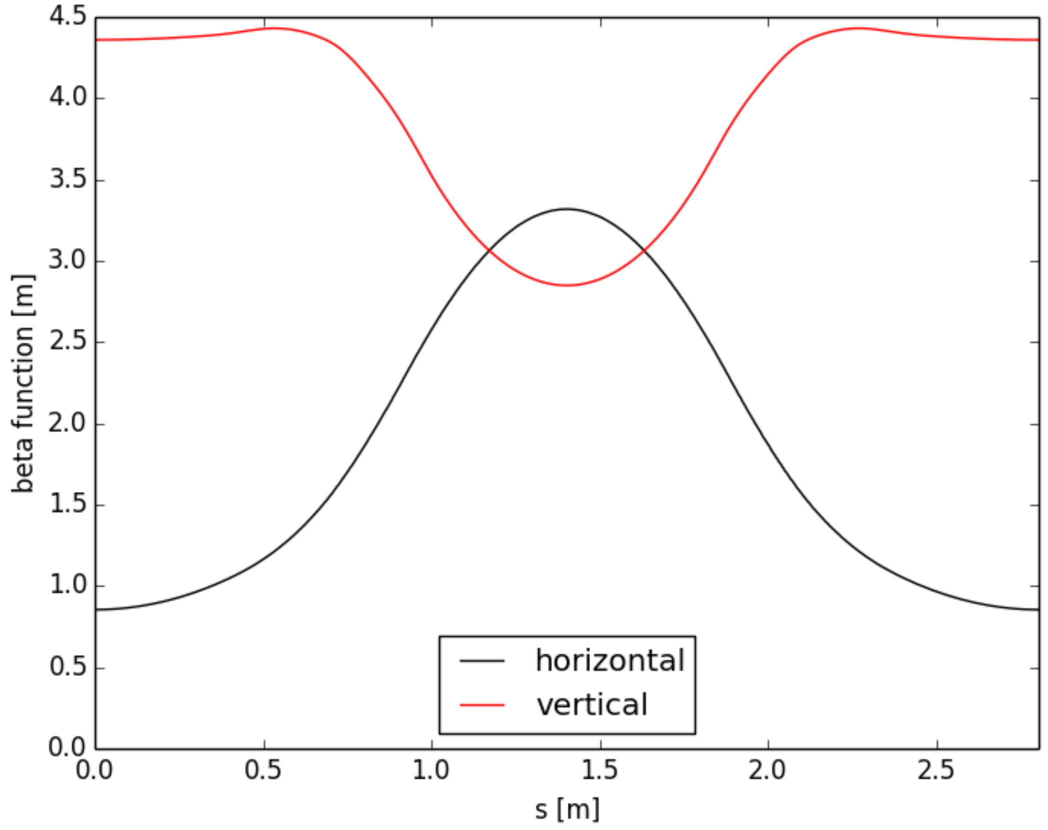
Test case - KURRI 150 MeV scaling FFAG



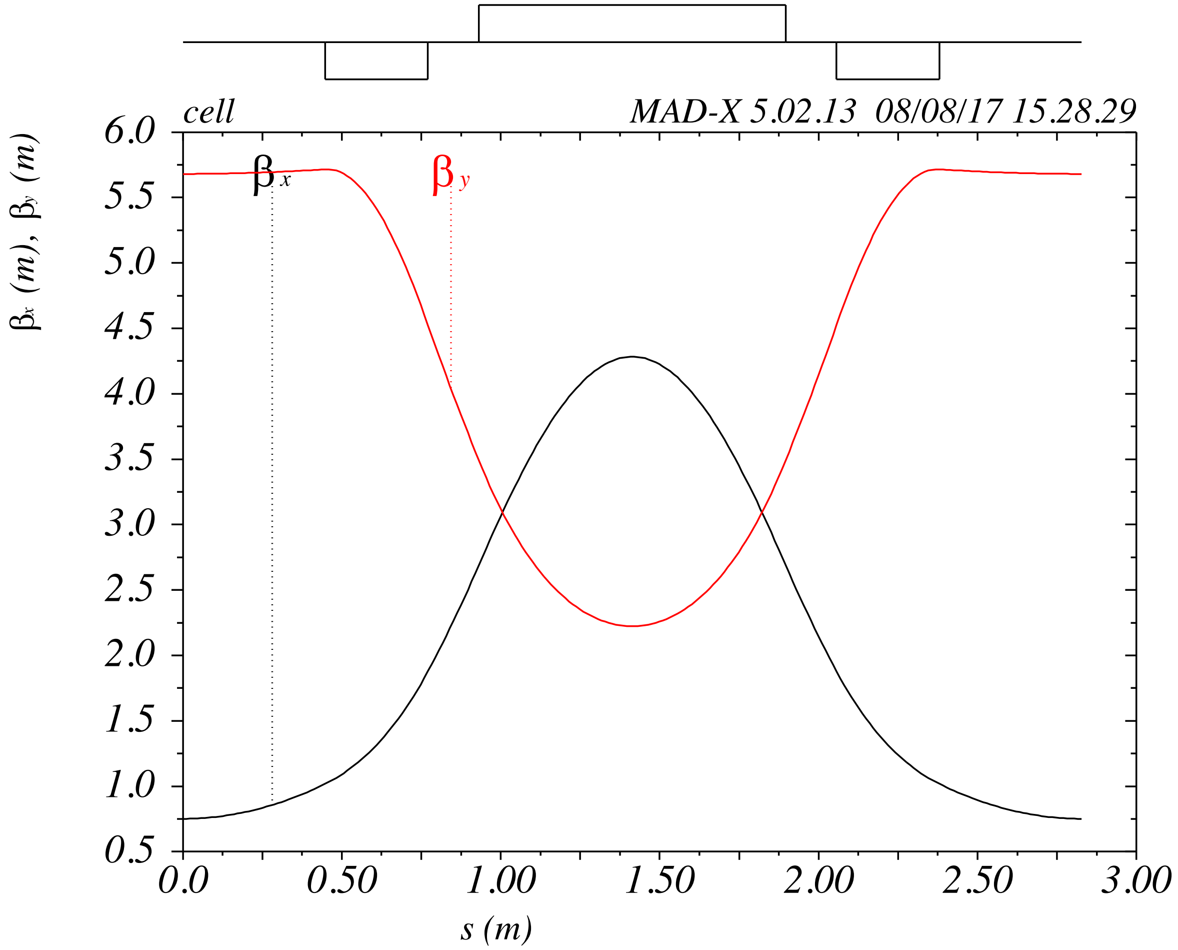
Machine parameters	
Reference radius	5.4
Focusing structure	DFD
Cell number	12
Injection energy	11 MeV
Extraction energy	100-150 MeV
Field index, κ	7.6
Design tunes (Qx,Qy)	(3.8, 1.4)

- Construct quadrupole triplet with lengths given by arc length of FFAG magnets
- Strength of quadrupole given by $\kappa/\rho r$, qf, qd = (1.3, -0.9). Vertical tune unstable!
- Instead use MADX optimisation to find qf, qd = (1.9, -2.3) results in desired tunes.

Step 1: Scaling FFAG to quadrupole channel

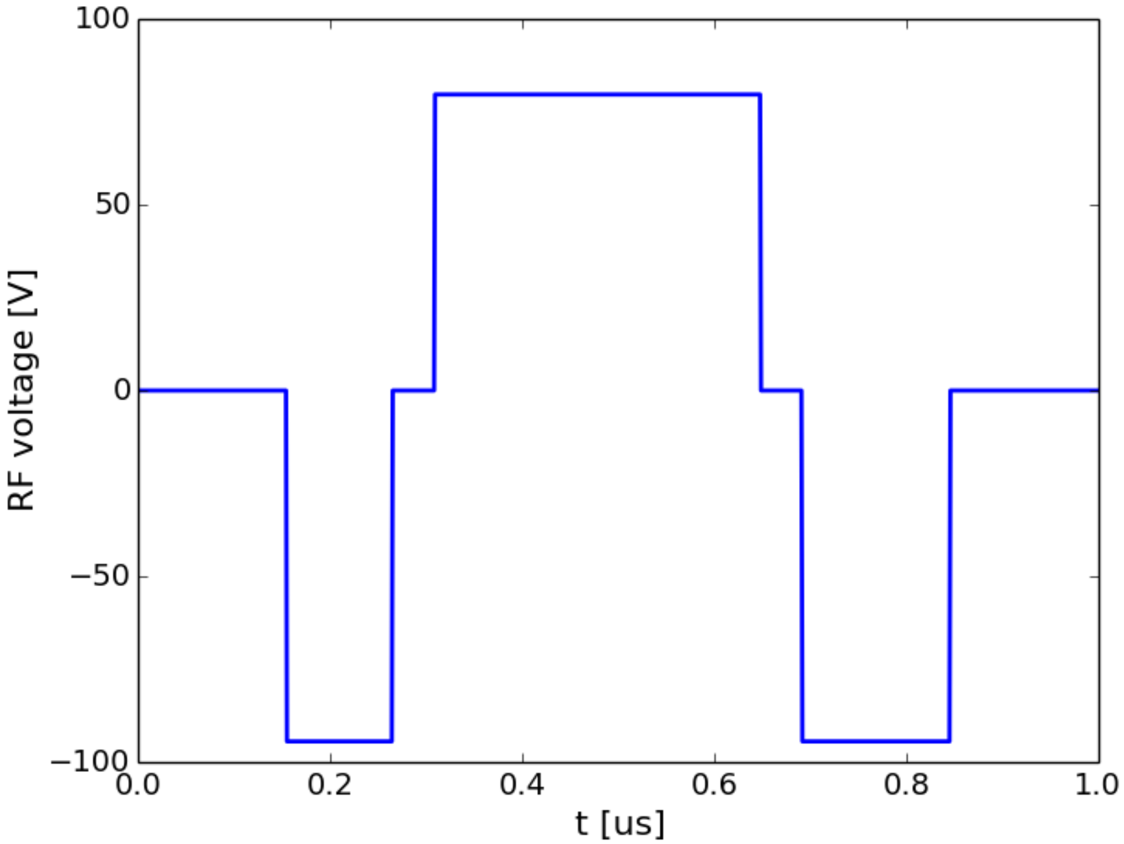


Zgoubi result, Qx, Qy = (0.314, 0.117).

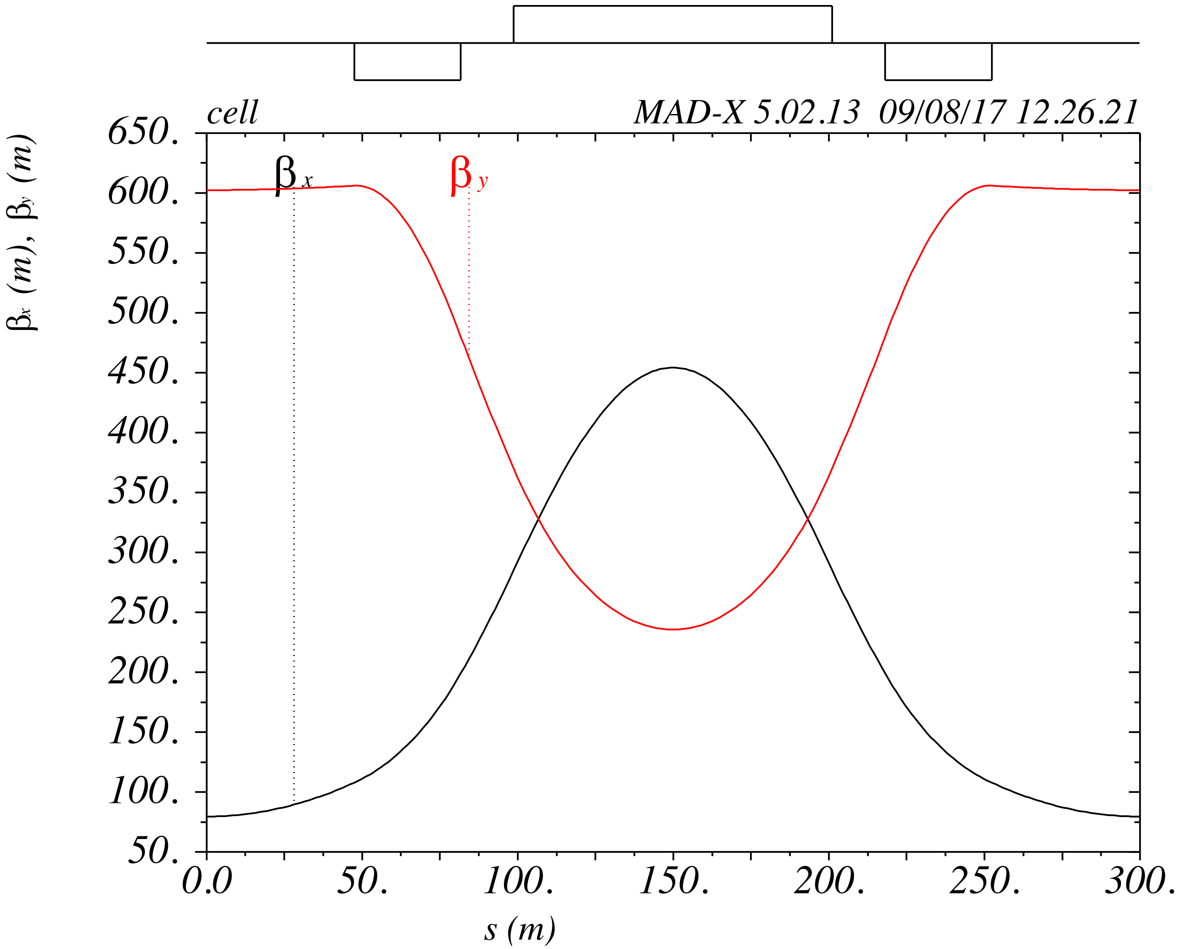


MADX, Qx, Qy = (0.314, 0.117).

Step 2: Quadrupole channel to LPT



RF waveform.



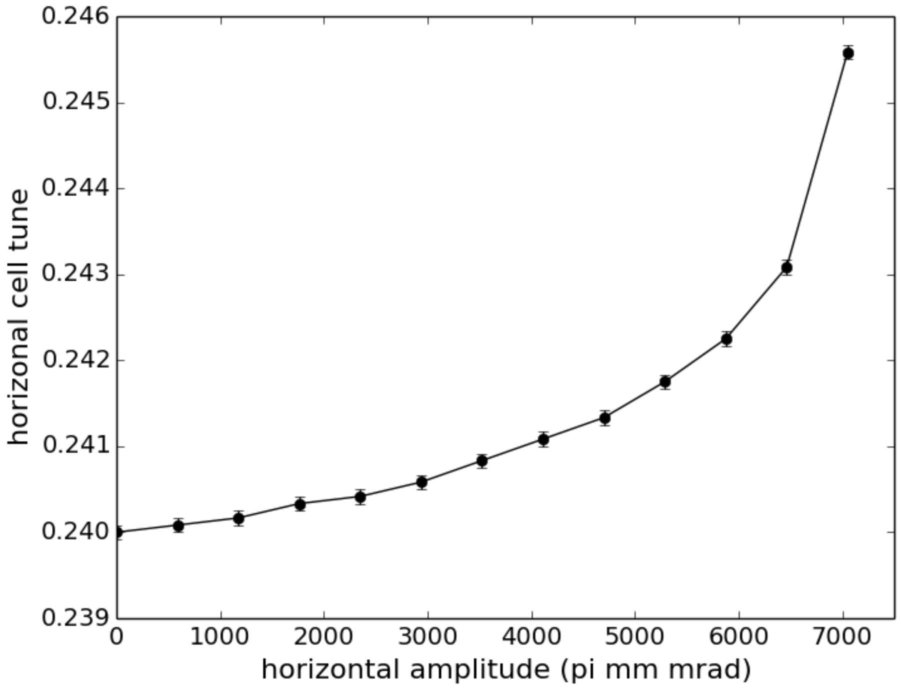
Equivalent trap optics where 1 "cell" is a 1us period.

Tunes Qx, Qy = (0.314, 0.117).

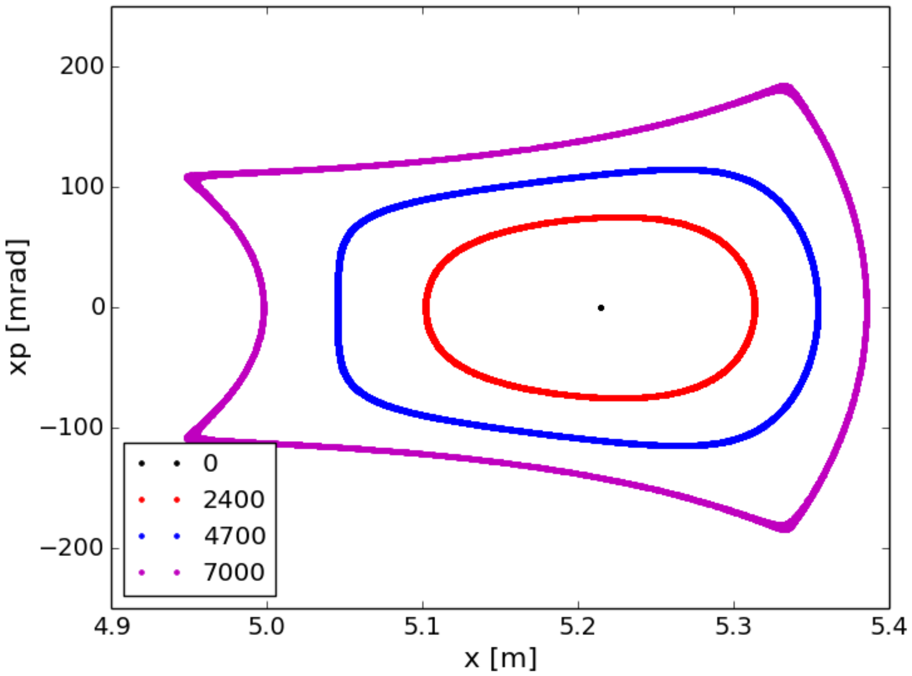
Example FFAG scenario

cell tune close to 1/4

Assume a ring with KURRI 150MeV geometry but adjust field index and F/D ratio to get desired tunes.

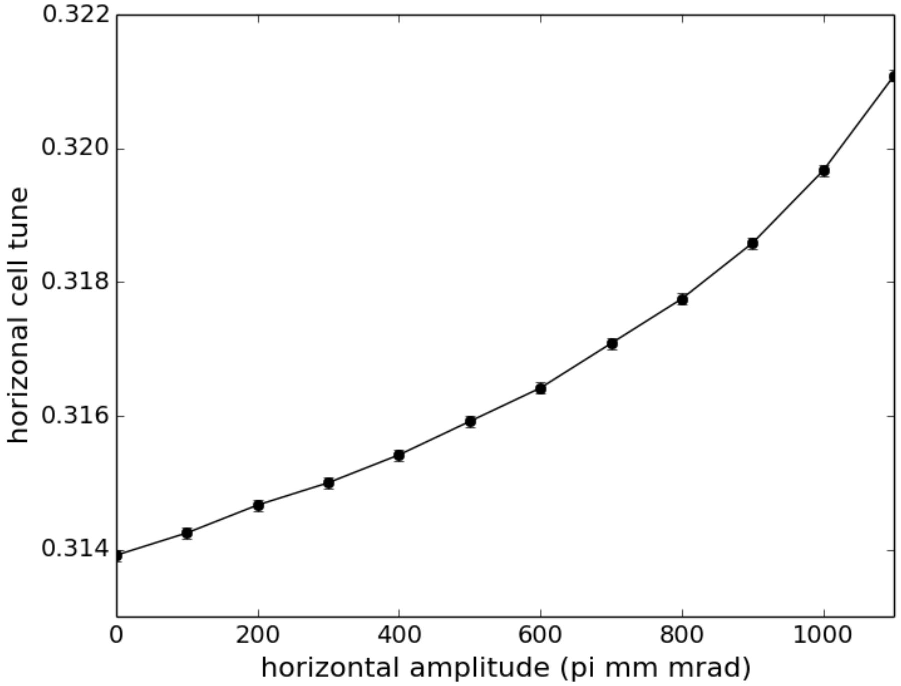


Tune variation with amplitude

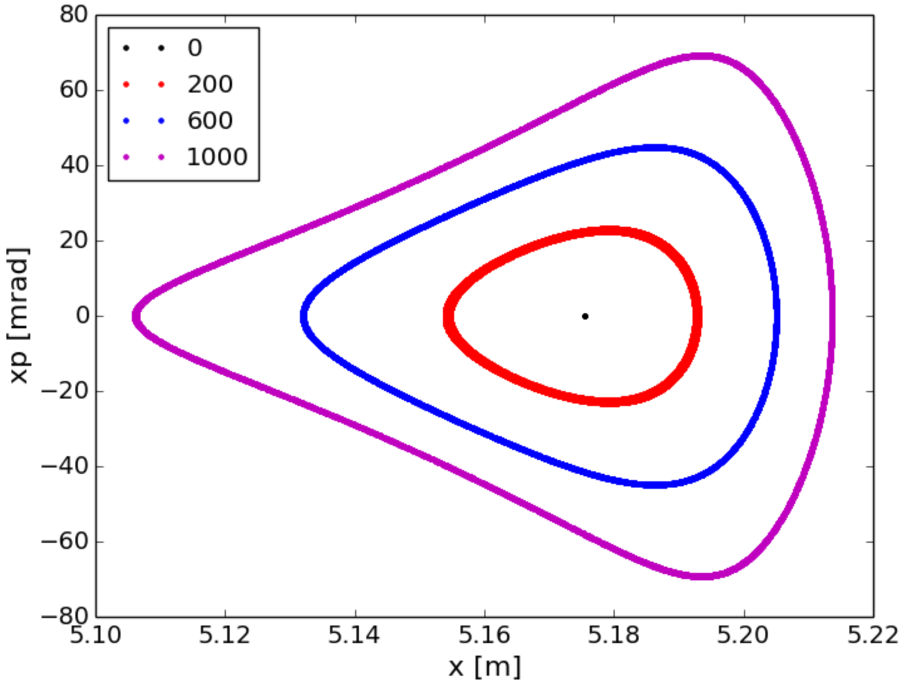


Phase space

Example FFAG scenario
cell tune close to 1/3

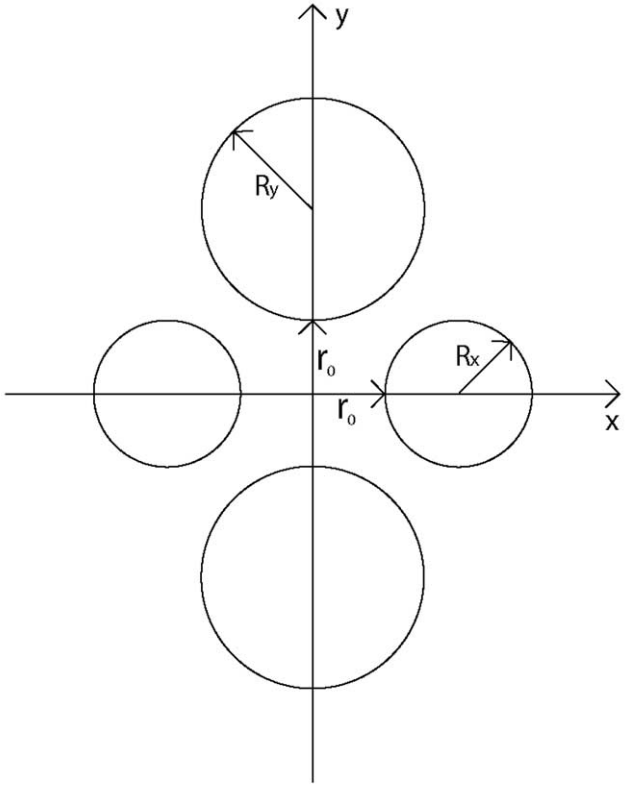


Tune variation with amplitude

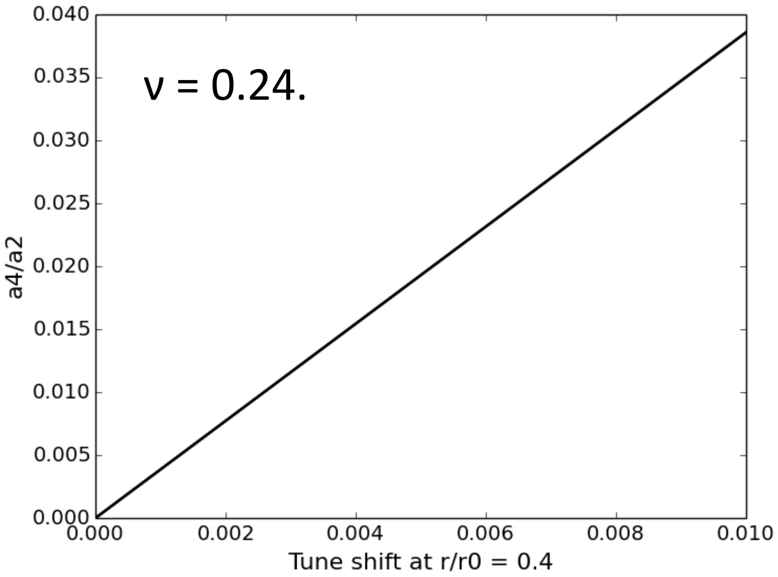


Phase space

Amplitude detuning in an asymmetric trap.



$$V(x,y) = \left[a_0 + a_2\left(\frac{x^2 - y^2}{r_0^2}\right) + a_4\left(\frac{x^4 - 6x^2y^2 + y^4}{r_0^4}\right)\right](U - V_{rf}\cos\Omega t)$$



$$\Delta\omega = 3\frac{a_4}{a_2}\left(\frac{r^2}{r_0^2}\right)\omega_0$$

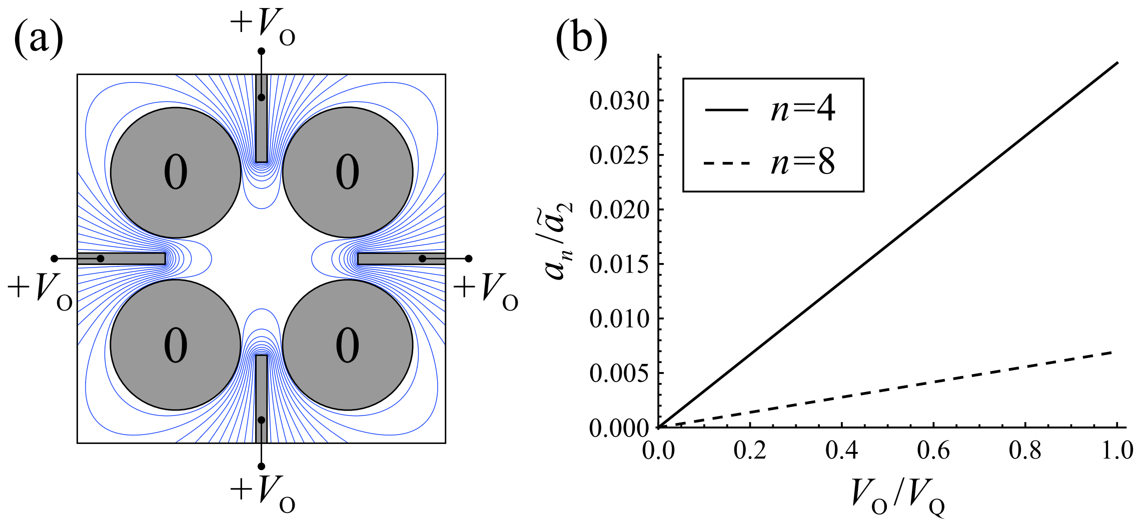
$$\downarrow$$

$$\frac{a_4}{a_2} = \frac{1}{3}\left(\frac{r_0}{r}\right)^2\frac{\Delta\nu}{\nu}$$

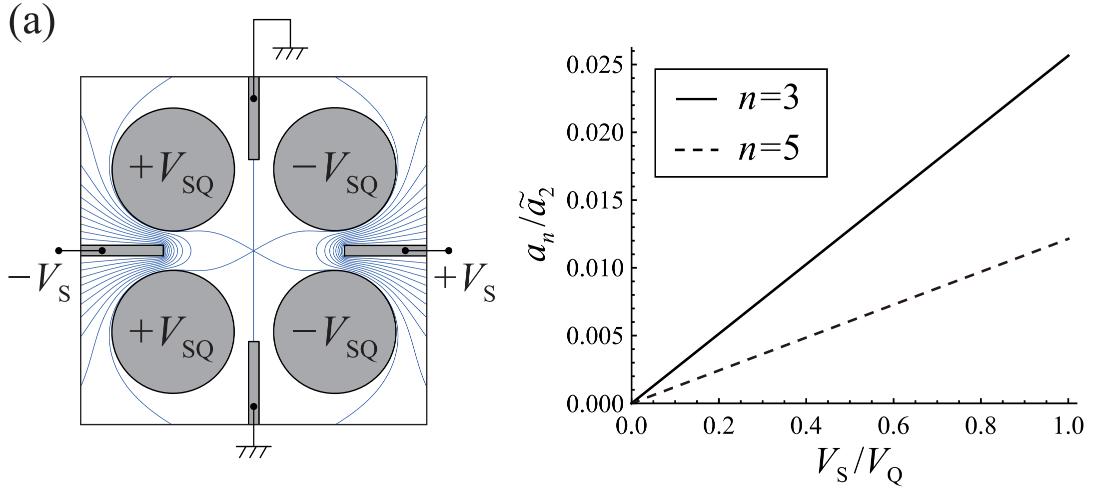
- One approach is to ensure the same tune variation from the beam centroid to the rms radius.
- However, detuning with amplitude (action) will be differ in Paul trap (β is much higher).

[Michaud et al, 2005]

K. Fukushima and H. Okamoto multipole trap design



Octupole mode



Sextupole mode

A multipole trap has been constructed and is now being commissioned.

K. Fukushima and H. Okamoto, ”Design study of a multipole ion trap for beam physics applications”, Plasma and Fusion Res. 10 (2015) 1401081.

Conclusions/Future work

- Experiments relevant to a scaling FFAG could be carried out in a multipole Paul trap with controllable sextupole and octupole.
- Further investigations required into traps with even higher order multipoles.
- 2D and 3D tracking of plasma in trap, including space charge, using a PIC code (e.g. Warp) is desirable.

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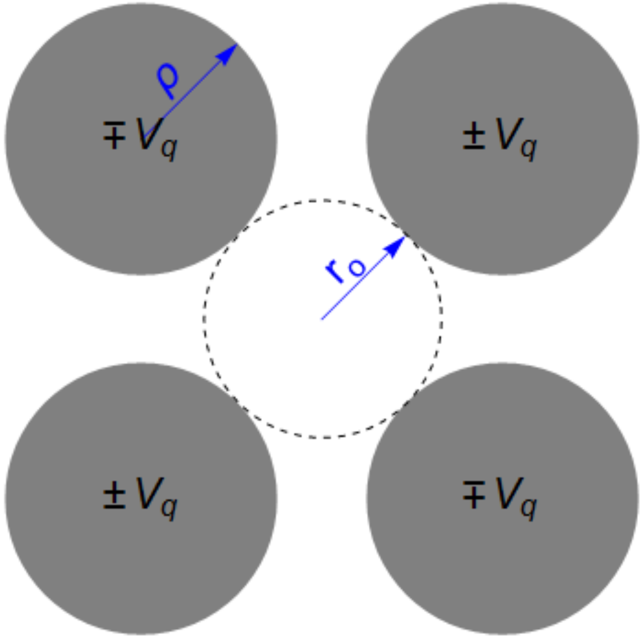
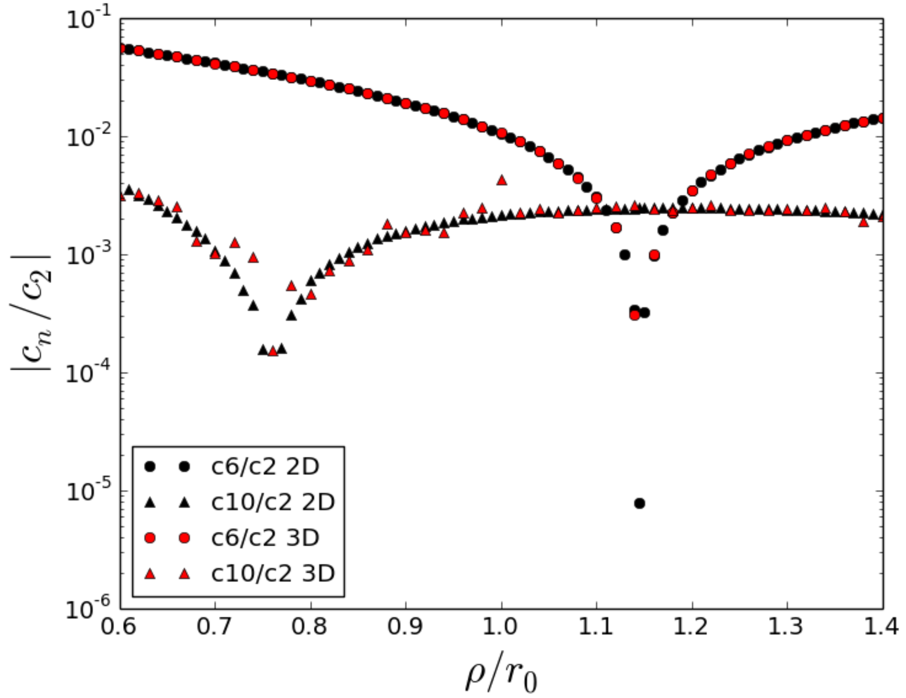
Extra Slides

Rod radius optimisation.





- Using cylindrical rods rather than ideal hyperbolae introduce a dodecapole component.
- This is minimised by choosing ρ/r0 ≅ 1.15.
- Results obtained using Mathematica (2D) and CST Studio (3D).

Square wave case.

- The equation of motion in the square waveform case is given by the Meissner equation which has an exact solution.

Paul trap case
$$\nu = \frac{1}{2\pi}\arccos\left[\cos(\pi\sqrt{q_i/2})\cosh(\pi\sqrt{q_i/2})\right]$$
[T. Brunner et al, 2012]

FODO case (zero drift length)
$$\nu = \frac{1}{2\pi}\arccos\left[\cos(\sqrt{k_2}L)\cosh(\pi\sqrt{k_2}L)\right]$$
Quadrupole length L, norm. gradient k_2.

Similarity of field profile

- In order to calculate multipole components, use Taylor expansion of the scaling field about the reference radius.

$$B = B_0 \left(\frac{r}{r_0}\right)^{\kappa} \longrightarrow B(r) = B_o \left[1 + \kappa \left(\frac{r - r_o}{r_o}\right) + \frac{\kappa(\kappa - 1)}{2} \left(\frac{r - r_o}{r_o}\right)^2 + \ldots \frac{\kappa!}{(n-1)!(\kappa - n + 1)!} \left(\frac{r - r_o}{r_o}\right)^{n-1}\right]$$

- In order to ensure a similar profile in the Paul trap, ensure that the ratio of the differential gradient for each multipole (n>2) and the quadrupole gradient is the same in both cases.
- In the scaling FFAG case the relative differential gradient is given by

$$\frac{1}{B'} \frac{d^{n-1} B^{(n)}}{dr^{n-1}} = \frac{(\kappa - 1)!}{(\kappa - n + 1)!} \frac{1}{r_o^{n-2}}$$

$$F(r, \theta) = \sum_{n=1}^{\infty} a_n \left(\frac{r}{r_0}\right)^n \cos(n\theta + \varphi_n) \longrightarrow \frac{1}{E'} \frac{d^{n-1} E^{(n)}}{dr^{n-1}} = \frac{(n+1)!}{2 r_0^{n-1}} \frac{a_n}{a_0}$$